Asymptotic Behavior of Nucleon-Nucleon Scattering*

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In this paper, a detailed analysis of NN and $N\bar{N}$ scattering on the basis of the Regge hypothesis is carried out. The Regge expansions of a set of ten invariant amplitudes describing NN scattering are presented, with residues expressed in factorized form. Expressions involving both the full Legendre functions and their asymptotic forms are given. Spin sums are carried out to obtain simple and convenient expressions for the contributions of the P, ρ, ω, and P' trajectories to the differential cross sections. The optical theorem has been applied to find the contribution of the P, P', ρ, and ω trajectories to the spin-averaged total cross sections. Finally, we have analyzed the available data on the total and differential cross sections for NN scattering to extract information about the Regge-pole parameters. The possible effect of the spin structure of the amplitudes, and the variation with energy of the Legendre functions has been taken into account. We show, by a natural definition of helicity flip and no-flip couplings, that the amplitudes, and especially the cross sections, for NN scattering are very simple in the asymptotic limit. In an Appendix, the decay properties of a spin-2 meson associated with the Pomeranchuk Regge pole are discussed.

I. INTRODUCTION

IN this paper we shall discuss nucleon-nucleon and nucleon-antinucleon scattering at high energies ($s\rightarrow\infty$) and low-momentum transfer $-s\ll t<0$. It is in this regime of momentum and energy that the Regge pole hypothesis, in terms of which we shall discuss NN and $N\bar{N}$ scattering, finds its most immediate application.

The general features of the nucleon-nucleon problem have already been discussed in terms of Regge poles.[1] Simple expressions have been obtained for various differential cross sections on the basis of an analysis which ignored the spin structure of the amplitudes. Perhaps the most characteristic result of such a simple Regge pole analysis, which should also come out of any more detailed Regge analysis, is the prediction of a diffraction cross section which, as energies become arbitrarily large, and momentum transfers remain small, has the functional form

$$\left(\frac{d\sigma}{dt}\right)\Big/\left(\frac{d\sigma}{dt}\right)_{t=0}=F(t)(s/s_0)^{2[\alpha(t)-1]}. \tag{1.1}$$

Recent data[2] on pp scattering in the range $15\lesssim s/2m_N^2\lesssim 25$, $0<-t/2m_N^2<3$ have been analyzed[3] in terms of Eq. (1.1), with the important result that at least the most general features of the Regge hypothesis (as applied to nucleon scattering) seem to be consistent with experiment.

The nucleon-nucleon system is of intrinsic importance in elementary particle and nuclear physics. The complicated spin structure of the amplitudes means that there will be many independent physical quantities in the NN and $N\bar{N}$ system which can be expressed in terms of Regge poles. With these, more detailed and precise experimental consequences of the Regge hypothesis can be deduced, and their investigation will lead to correspondingly more stringent tests of the Regge hypothesis. In terms of experimental feasibility, the nucleon-nucleon system appears to be the most suitable for further detailed experimental verification of the Regge pole conjecture. For all these reasons, we feel that the nucleon-nucleon system merits a thorough treatment based on the Regge pole hypothesis, which is given in this paper.

Consequently, we present in Sec. IIA the leading terms in the Regge expansions of a set of ten invariant amplitudes, which are free of kinematic singularities, describing NN and $N\bar{N}$ scattering. We discuss the possible transitions in NN and $N\bar{N}$ scattering between states of given parity, spin, and isospin. These are conveniently summarized in terms of τP [= (signature)(parity)] and $(-)^I GP$. The selection rules which result reduce the number of independent amplitudes describing the scattering which arises from a given Regge pole. Regge expansions for the helicity amplitudes are also obtained in this section. In Sec. IIB we briefly discuss a high-energy symmetry between reactions whose amplitudes are related by a reversal of external baryon lines.[4,5]

The expansions we derive in this section are of interest regardless of whether the set of important singularities in the angular-momentum plane consists of poles only or contains also cuts. However, the usefulness of the Regge asymptotic expansion for data analysis will be seriously imparied if cuts play a very significant role.

The functions $b_i(u)$ occurring in the Regge expansions are related to certain coupling strengths. In Sec. III we establish the precise relationships in a number of par-

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[1] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).
[2] A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **9**, 32 (1962).
[3] A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **9**, 108, 111 (1962).
[4] M. Gell-Mann, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 533.
[5] William G. Wagner and David H. Sharp, Phys. Rev. **128**, 2899 (1962).

ticular cases by comparing the Regge amplitude to the corresponding Feynman amplitude at the pole.

We should like to mention at this point that other discussions of the Regge expansions of the NN and $N\bar{N}$ amplitudes have also been carried out,[4,6–8] and some of the results of Sec. II of our article are contained in these papers. In particular, Gell-Mann[4] has presented his expressions for the amplitude in "factorized" form, as shall also be done in this paper. In addition, he has analyzed in a most interesting way the question of the presence of "ghosts" in these amplitudes. Muzinich,[8] in his discussion of the Regge expansions of NN and $N\bar{N}$ amplitudes, considers a problem not discussed here; namely, he shows (on the basis of the Mandelstam representation) that the Froissart[9] analytic continuation of the partial-wave helicity amplitudes can be carried out for the NN problem, where the particles are spinors.

In Sec. IV we discuss in detail the cross sections for NN and $N\bar{N}$ scattering.[10] The contributions of the P, ρ, ω, and P' trajectories are all discussed. All spin sums are carried out explicitly.

In Sec. V we turn to an analysis of existing data on NN and $N\bar{N}$ scattering in terms of the Regge pole hypothesis. Our analysis is based on the data of Diddens *et al.*,[2,3] and of Lindenbaum *et al.*[11] We find that an analysis which includes the full variation of the Legendre functions with energy, as well as the spin structure of the amplitudes, does not change the basic conclusions[12,13] of the Regge analysis of total cross sections. A second vacuum trajectory, introduced by K. Igi,[14] is consistent with the data. However, because the $\sigma_{p\bar{p}}$ data[11] are so far from satisfying the Pomeranchuk theorem, and because the σ_{np} data, containing the Glauber correction, are so unreliable, the conclusions of such an analysis must be regarded as rather tentative.

The angular distributions have been expressed in terms of the Regge-pole parameters. If only the Pomeranchuk trajectory is included, the differential cross sections can be expressed in terms of essentially one function, a result which becomes clear when the differential cross sections are expressed in terms of helicity amplitudes.[15] The available data have been used to determine this function; we find it has a linear behavior for $0<-t\lesssim 0.40$ (GeV)^2, and is a constant in this region if $s_0=1$ (GeV)^2.

Throughout this paper, our emphasis has been on exploring the detailed experimental consequences of the Regge hypothesis as applied to the nucleon-nucleon system. It is hoped that this effort will instigate more elaborate experimental investigations, designed to test critically the predictions made here. We wish to check, as thoroughly as possible by experiment, whether this approach to elementary particle physics has a firm basis in the facts of nature.

II. PROPERTIES OF THE AMPLITUDES DESCRIBING NUCLEON-NUCLEON SCATTERING

A. Regge Expansions for Nucleon-Nucleon Scattering Amplitudes

The Regge-pole contributions to the amplitude may be deduced from the partial-wave expansion of the amplitude in the cross channel according to the prescription of Frautschi, Gell-Mann, and Zachariasen.[1,4] To obtain them, we may employ the matrices of Goldberger, Grisaru, MacDowell, and Wong[16] who have discussed the application of the Mandelstam representation to the NN problem. GGMW, and Amati, Leader, and Vitale[17] have shown that only if the NN scattering amplitude is expressed in terms of Fermi invariants are the associated invariant functions free of kinematic singularities.

In order to facilitate comparison with previous work, we shall adopt the notation introduced by GGMW. The nucleon-nucleon scattering amplitude is written as

$$T=\sum_I \beta^I[F_S{}^I(s,u,t)S+F_T{}^I(s,u,t)T+F_A{}^I(s,u,t)A+F_V{}^I(s,u,t)V+F_P{}^I(s,u,t)P],$$

where

$$\begin{aligned}
S&=\bar{u}(p_1')1u(p_1)\bar{u}(p_2')1u(p_2),\\
T&=\tfrac{1}{2}\bar{u}(p_1')\sigma_{\mu\nu}u(p_1)\bar{u}(p_2')\sigma_{\mu\nu}u(p_2),\\
A&=\bar{u}(p_1')i\gamma_5\gamma_\mu u(p_1)\bar{u}(p_2')i\gamma_5\gamma_\mu u(p_2), \qquad (2.0a)\\
V&=\bar{u}(p_1')\gamma_\mu u(p_1)\bar{u}(p_2')\gamma_\mu u(p_2),\\
P&=\bar{u}(p_1')\gamma_5 u(p_1)\bar{u}(p_2')\gamma_5 u(p_2),
\end{aligned}$$

$$\beta^1=\tfrac{1}{4}[(\bar{s}_{1'}\boldsymbol{\tau}s_1)\cdot(\bar{s}_{2'}\boldsymbol{\tau}s_2)+3(\bar{s}_{1'}s_1)(\bar{s}_{2'}s_2)], \qquad (2.0b)$$

$$\beta^0=\tfrac{1}{4}[-(\bar{s}_{1'}\boldsymbol{\tau}s_1)\cdot(\bar{s}_{2'}\boldsymbol{\tau}s_2)+(\bar{s}_{1'}s_1)(\bar{s}_{2'}s_2)], \qquad (2.0c)$$

and

$$\begin{aligned}
s&=-(p_1+p_2)^2,\\
u&=-(p_2'-p_1)^2, \qquad (2.0d)\\
t&=-(p_1'-p_1)^2.
\end{aligned}$$

In the above, the s_i represent isospinors.

The inclusion of the isotopic factors, which we usually drop for the sake of simplicity, is accomplished quite

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[7] Y. Hara, Phys. Letters **2**, 246 (1962); and Progr. Theoret. Phys. (Kyoto) **28**, 711 (1962).

[8] I. J. Muzinich, Ph.D. thesis, University of California, Berkeley, UCRL-10331 (unpublished); Phys. Rev. Letters **9**, 475 (1962).

[9] M. Froissart, Report at the La Jolla Conference on Weak and Strong Interactions, 1961 (unpublished); also Phys. Rev. **123**, 1053 (1961).

[10] In this connection, see also Ref. 7.

[11] S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **7**, 185 (1961).

[12] S. Drell, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 897.

[13] F. Hadjioannou, R. J. N. Phillips, and W. Rarita, Phys. Rev. Letters **9**, 183 (1962).

[14] K. Igi, Phys. Rev. Letters **9**, 76 (1962).

[15] William G. Wagner, Phys. Rev. Letters **10**, 202 (1963).

[16] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960). This paper will be referred to hereafter as GGMW.

[17] D. Amati, E. Leader, and B. Vitale, Nuovo Cimento **17**, 68 (1960). This paper will be referred to hereafter as ALV.

readily by making use of the matrix

$$(\Lambda_{II'})=\frac{1}{2}\begin{pmatrix}1 & -3\\ 1 & 1\end{pmatrix} \tag{2.0e}$$

which relates the invariant functions F^I, considered as a two-component vector in the isotopic spin index, to the relevant functions in the t channel with isospin I'. The first row and column refer to $I=0$, and the second to $I=1$.

Throughout this paper, we shall suppose that the Regge pole is in the t channel. However, we may briefly indicate here how to pass from the t channel to the u channel, or vice versa ($t\rightleftarrows u$). This corresponds to interchanging p_1' and p_2'. The following changes are thereby produced: (i) The full amplitude changes sign; (ii) the spinors of the final particles are interchanged, $u(p_1')\rightleftarrows u(p_2')$; (iii) in the c.m. system, the scattering angle changes from θ to $\pi-\theta$; (iv) finally, the isospin projection operator β^0 changes sign while β^1 does not. The matrix (2.0e) then becomes

$$(\Lambda_{II'})=-\frac{1}{2}\begin{pmatrix}-1 & 3\\ 1 & 1\end{pmatrix}. \tag{2.0e$'$}$$

It should be noticed that Eq. (2.0e′) already includes the sign change mentioned in (i) above.

The first part of the problem is to ascertain the contributions to the five invariant functions, F_S, F_T, F_A, F_V, and F_P, resulting from a Regge pole characterized by definite values of G, I, P, and signature (τ). This may be expedited by employing some of the formulas of GGMW. (In the following kinematic considerations, we shall omit the isospin factor.) With the aid of Eq. (2.6) of GGMW, we see that

$$\begin{bmatrix}\tilde{S}-S\\ \tilde{T}+T\\ \tilde{A}-A\\ \tilde{V}+V\\ \tilde{P}-P\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-3 & 1 & 1 & 1 & 1\\ 6 & 2 & 0 & 0 & 6\\ 4 & 0 & -6 & 2 & -4\\ 4 & 0 & 2 & 2 & -4\\ 1 & 1 & -1 & -1 & -3\end{bmatrix}\begin{bmatrix}S\\ T\\ A\\ V\\ P\end{bmatrix}, \tag{2.1}$$

and, consequently,

$$\begin{bmatrix}F_S\\ F_T\\ F_A\\ F_V\\ F_P\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-3 & 6 & 4 & 4 & 1\\ 1 & 2 & 0 & 0 & 1\\ 1 & 0 & -6 & 2 & -1\\ 1 & 0 & 2 & 2 & -1\\ 1 & 6 & -4 & -4 & -3\end{bmatrix}\begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\\ F_5\end{bmatrix}, \tag{2.2}$$

where

$$T=F_1(\tilde{S}-S)+F_2(\tilde{T}+T)+F_3(\tilde{A}-A)+F_4(\tilde{V}+V)+F_5(\tilde{P}-P).$$

The set of invariant functions $\{F_1,F_2,F_3,F_4,F_5\}$ have nice symmetries under the interchange $u\leftrightarrow t$ due to the generalized Pauli principle, but in the Regge pole considerations, it is much more convenient to work with our unsymmetrized functions. Inverting Eq. (4.24) of GGMW, we have

$$\begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\\ F_5\end{bmatrix}=\frac{1}{2\pi}\begin{bmatrix}1 & 0 & 4 & 0 & 3\\ 0 & 4 & 0 & 0 & 0\\ -1 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 4 & 0\\ 3 & 0 & -4 & 0 & 1\end{bmatrix}\begin{bmatrix}G_1\\ G_2\\ G_3\\ G_4\\ G_5\end{bmatrix}, \tag{2.3}$$

and using Eqs. (4.27) and (4.28) of GGMW to relate G_i and $\bar{G}_i$, we obtain

$$\begin{bmatrix}F_S\\ F_T\\ F_A\\ F_V\\ F_P\end{bmatrix}=\frac{1}{2\pi}\begin{bmatrix}0 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\bar{G}_1\\ \bar{G}_2\\ \bar{G}_3\\ \bar{G}_4\\ \bar{G}_5\end{bmatrix}. \tag{2.4}$$

Thus, the $\bar{G}_i$ of GGMW are exactly the same as the choice of invariant functions convenient for our analysis.

The partial-wave decomposition of the G's may be obtained by using Eq. (4.33) of GGMW, which in our notation reads

$$\bar{G}(\bar{f})=\begin{bmatrix}1/E^2 & 0 & m^2/E^2p^2 & -z/E^2 & -z/m^2\\ 0 & 0 & 0 & -1/p^2 & -E^2/m^2p^2\\ 0 & 0 & -1/p^2 & 0 & 0\\ 0 & 0 & 0 & 1/p^2 & 1/p^2\\ 0 & -1/p^2 & 0 & -z/p^2 & -z(E^2+m^2)/m^2p^2\end{bmatrix}, \tag{2.5}$$

where $4p^2=t-4m^2$, $4E^2=t$, and $z=-[1+2s/(t-4m^2)]$, together with equations which relate the f_i to their partial-wave forms.

The angular functions employed for this purpose were evaluated from the reduction formulas of Jacob and Wick[18] with the following results:

$$f_1=(E/p)(2J+1)P_J(z)\bar{f}_0{}^J, \tag{2.6a}$$

$$f_2=(E/p)(2J+1)P_J(z)\bar{f}_{11}{}^J, \tag{2.6b}$$

$$f_5=-(m/p)(2J+1)P_J'(z)\bar{f}_{12}{}^J/[J(J+1)]^{1/2}, \tag{2.6c}$$

$$\bar{f}_3=\frac{E}{p}\frac{(2J+1)}{J(J+1)}\left\{\left[P_J'+\frac{z(2P_{J-1}'-J(J-1)P_J)}{1-z^2}\right]\bar{f}_1{}^J-\left[\frac{2P_{J-1}'-J(J-1)P_J}{1-z^2}\right]\bar{f}_{22}{}^J\right\}, \tag{2.6d}$$

$$\bar{f}_4=f_3(\bar{f}_1{}^J\leftrightarrow\bar{f}_{22}{}^J). \tag{2.6e}$$

We can now easily obtain the Regge amplitudes corre-

[18] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959).

sponding to a given trajectory. Use of Eqs. (2.4) and (2.5) can be made to obtain the partial-wave expansions of the amplitudes $F_S, \cdots, F_P$. These are summarized in Table I for states of the $N\bar{N}$ system classified by the quantum numbers τP, J, and $(-)^I GP$.

Mandelstam[19] has shown that it is likely that the true asymptotic expansion of the amplitudes in the sense of Regge involves Legendre functions of the second kind rather than those of the first kind. The transition from the Regge expansion to the modified expansion amounts to the replacement[4] of $P_\alpha(x)$ by $\mathcal{P}_\alpha(x)$, where $\mathcal{P}_\alpha(x)=-[\tan\pi\alpha Q_{-\alpha-1}(x)]/\pi$. In this paper we shall write the expansion formally in terms of the P_α for typographical reasons only; in practice, it makes no difference in the data analysis whether one uses P_α or the more correct $\mathcal{P}_\alpha$.

The next step is to factor out the threshold behavior in the functions f. We may do so by introducing the functions b_i according to the definitions:

$$f_{11}{}^\alpha(t)=\left[-\frac{\alpha!\sqrt{\pi}}{2^{\alpha+1}(\alpha+\frac{1}{2})!}\left(\frac{p^3}{2\pi E}\right)\left(\frac{2s_0}{4m^2}\right)\right.$$
$$\left.\times\left(\frac{t-4m^2}{2s_0}\right)^\alpha\right]\left[\frac{1+\tau e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}\right]b_{11}(t)\,, \quad (2.7a)$$

$$\frac{\bar{f}_{12}{}^\alpha(t)}{[\alpha(\alpha+1)]^{1/2}}=\frac{E}{m}\left[-\frac{\alpha!\sqrt{\pi}}{2^{\alpha+1}(\alpha+\frac{1}{2})!}\left(\frac{p^3}{2\pi E}\right)\left(\frac{2s_0}{4m^2}\right)\right.$$
$$\left.\times\left(\frac{t-4m^2}{2s_0}\right)^\alpha\right]\left[\frac{1+\tau e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}\right]b_{12}(t)\,, \quad (2.7b)$$

$$\frac{f_{22}{}^\alpha(t)}{\alpha(\alpha+1)}=\frac{E^2}{m^2}\left[-\frac{\alpha!\sqrt{\pi}}{2^{\alpha+1}(\alpha+\frac{1}{2})!}\left(\frac{p^3}{2\pi E}\right)\left(\frac{2s_0}{4m^2}\right)\right.$$
$$\left.\times\left(\frac{t-4m^2}{2s_0}\right)^\alpha\right]\left[\frac{1+\tau e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}\right]b_{22}(t)\,, \quad (2.7c)$$

$$f_0{}^\alpha(t)=\frac{E^2}{p^2}\left[-\frac{\alpha!\sqrt{\pi}}{2^{\alpha+1}(\alpha+\frac{1}{2})!}\left(\frac{p^3}{2\pi E}\right)\left(\frac{2s_0}{4m^2}\right)\right.$$
$$\left.\times\left(\frac{t-4m^2}{2s_0}\right)^\alpha\right]\left[\frac{1+\tau e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}\right]b_0(t)\,, \quad (2.7d)$$

$$\frac{f_1{}^\alpha(t)}{\alpha(\alpha+1)}=\left(\frac{4m^2}{t-4m^2}\right)\frac{t}{4m^2}$$
$$\times\left[-\frac{\alpha!\sqrt{\pi}}{2^{\alpha+1}(\alpha+\frac{1}{2})!}\left(\frac{p^3}{2\pi E}\right)\left(\frac{2s_0}{4m^2}\right)\right.$$
$$\left.\times\left(\frac{t-4m^2}{2s_0}\right)^\alpha\right]\left[\frac{1+\tau e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}\right]b_1(t)\,. \quad (2.7e)$$

The new expressions for the amplitudes in Table I may be written conveniently in terms of the b_i and[4]

$$Z_\alpha{}^\tau(s,t)=[Z_\alpha(s,t)+\tau Z_\alpha(u,t)]/(1+\tau e^{-i\pi\alpha(t)})\,,$$

where

$$Z_\alpha(s,t)\equiv e^{-i\pi\alpha}\frac{\alpha!\sqrt{\pi}}{2^\alpha(\alpha-\frac{1}{2})!}\left(\frac{t-4m^2}{2s_0}\right)^\alpha$$
$$\times P_\alpha\left[-\left(1+\frac{2s}{t-4m^2}\right)\right]. \quad (2.8)$$

The asymptotic behavior of these functions is independent of the signature:

$$Z_\alpha(s,t)=\left(\frac{2s+t-4m^2}{2s_0}\right)^\alpha$$
$$-\frac{\alpha(\alpha-1)(t-4m^2)^2}{2(2\alpha-1)4s_0{}^2}\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha-2}+\cdots$$
$$=\left(\frac{2s+t-4m^2}{2s_0}\right)^\alpha\left[1-\frac{\alpha(\alpha-1)}{2(2\alpha-1)}x^2+\cdots\right], \quad (2.9)$$

where $x=(t-4m^2)/(2s+t-4m^2)$, and

$$Z_\alpha{}'(s,t)=dZ_\alpha/d(s/s_0)$$
$$=\alpha[(2s+t-4m^2)/2s_0]^{\alpha-1}[1-\cdots]. \quad (2.10)$$

Upon substituting the Z_α into the amplitudes of Table I, we obtain formulas for the Regge pole terms in the NN-scattering amplitude.

Thus far in our analysis, we have not incorporated the hypothesis that the Regge pole terms are factorizable.[20] The effect of this property is to reduce the number of independent invariant functions, $b_i(t)$, from three to two in the case where the Regge trajectory has the quantum number $\tau P=+$. It results in no change for those contributions to the invariant functions arising from trajectories with $\tau P=-$, since there is only one invariant function, $b_0(t)$ or $b_1(t)$, associated with such poles.

The relations

$$tF_P+4m^2F_A+(2s+t-4m^2)F_T=0\,,$$

and

$$(tF_V+4m^2F_T)[2P_{\alpha-1}{}'(-1-2s/(t-4m^2))-\alpha(\alpha-1)P_\alpha]$$
$$+F_A(4m^2-t)[(1-z^2)P_\alpha{}'+z(2P_{\alpha-1}{}'-\alpha(\alpha-1)P_\alpha)]=0,$$

are valid for the contributions from Regge poles with $\tau P=+$, irrespective of whether the coupling to the pole may be factorized. The additional relation imposed by the factorizability of the pole contribution, however, may not be expressed in the simple form of a linear relation between invariant functions. Rather, it leads to expressions for all the invariant functions as a bilinear form in two functions instead of as a linear form in three.

[19] S. Mandelstam, Ann. Phys. (N. Y.) **19**, 254 (1962).

[20] M. Gell-Mann, Phys. Rev. Letters **8**, 263 (1962); V. N. Gribov and I. Ya. Pomeranchuk, *ibid.* **8**, 346 (1962).

Table I. Partial-wave expansions of the invariant NN scattering amplitudes associated with the exchange of an object in the t channel with quantum numbers τP, J, and $(-)^I GP$.

Quantum numbers of object exchanged τP	J	$(-)^I GP$	Partial-wave expansion of amplitudes F_S, F_T, F_A, F_V, F_P
$+$	α	$+$	$F_S=-\dfrac{2\pi E(2\alpha+1)}{p^3}\left\{\bar{f}_{11}{}^{\alpha}P_\alpha(z)+z\left[P_\alpha'+\dfrac{z(2P_{\alpha-1}'-\alpha(\alpha-1)P_\alpha)}{1-z^2}\right]\dfrac{\bar{f}_{22}{}^{\alpha}}{\alpha(\alpha+1)}-z(E^2/m^2+1)\dfrac{m}{E}\dfrac{P_\alpha'}{[\alpha(\alpha+1)]^{1/2}}\bar{f}_{12}{}^{\alpha}\right\}$
			$F_T=-\dfrac{2\pi E(2\alpha+1)}{p^3}\left\{\left[P_\alpha'+\dfrac{z(2P_{\alpha-1}'-\alpha(\alpha-1)P_\alpha)}{1-z^2}\right]\dfrac{\bar{f}_{22}{}^{\alpha}}{\alpha(\alpha+1)}-\dfrac{E}{m}\dfrac{P_\alpha'}{[\alpha(\alpha+1)]^{1/2}}\bar{f}_{12}{}^{\alpha}\right\}$
			$F_A=\dfrac{2\pi E(2\alpha+1)}{p^3}\dfrac{[2P_{\alpha-1}'-\alpha(\alpha-1)P_\alpha]}{1-z^2}\dfrac{\bar{f}_{22}{}^{\alpha}}{\alpha(\alpha+1)}$
			$F_V=\dfrac{2\pi E(2\alpha+1)}{p^3}\left\{\left[P_\alpha'+\dfrac{z(2P_{\alpha-1}'-\alpha(\alpha-1)P_\alpha)}{1-z^2}\right]\dfrac{\bar{f}_{22}{}^{\alpha}}{\alpha(\alpha+1)}-\dfrac{m}{E}\dfrac{P_\alpha'}{[\alpha(\alpha+1)]^{1/2}}\bar{f}_{12}{}^{\alpha}\right\}$
			$F_P=-\dfrac{2\pi(2\alpha+1)}{pE}\left\{\left[zP_\alpha'+\left(\dfrac{m^2}{p^2}+z^2\right)\dfrac{(2P_{\alpha-1}'-\alpha(\alpha-1)P_\alpha)}{1-z^2}\right]\dfrac{\bar{f}_{22}{}^{\alpha}}{\alpha(\alpha+1)}-z\dfrac{E}{m}P_\alpha'\dfrac{\bar{f}_{12}{}^{\alpha}}{[\alpha(\alpha+1)]^{1/2}}\right\}$
$-$	α	$-$	$F_S=F_T=F_A=F_V=0$
			$F_P=[2\pi(2\alpha+1)/pE]P_\alpha(z)\bar{f}_0{}^{\alpha}(t)$
$-$	α	$+$	$F_S=\dfrac{2\pi E(2\alpha+1)}{p^3}\dfrac{z}{1-z^2}[2P_{\alpha-1}'-\alpha(\alpha-1)P_\alpha]\dfrac{\bar{f}_1{}^{\alpha}}{\alpha(\alpha+1)}$
			$F_T=F_S/z$
			$F_A=-\dfrac{2\pi E(2\alpha+1)}{p^3}\left[P_\alpha'+\dfrac{z(2P_{\alpha-1}'-\alpha(\alpha-1)P_\alpha)}{1-z^2}\right]\dfrac{\bar{f}_1{}^{\alpha}}{\alpha(\alpha+1)}$
			$F_V=-F_T$
			$F_P=-(m^2/E^2)F_A+(p^2/E^2)F_S$

The functions $\bar{f}_{11}{}^J(t)$, $\bar{f}_{12}{}^J(t)$, and $\bar{f}_{22}{}^J(t)$ of GGMW, which appear first in our Eq. (2.6) are the elements of a 2×2 symmetric reaction matrix. The assumption that it may be factorized is equivalent to choosing the representation:

$$\bar{f}_{11}{}^J(t)=[\bar{f}_{1+}{}^J(t)]^2,\quad \bar{f}_{12}{}^J(t)=\bar{f}_{1+}{}^J(t)\bar{f}_{2+}{}^J(t),$$
$$\bar{f}_{22}{}^J(t)=[\bar{f}_{2+}{}^J(t)]^2.$$

It is natural, therefore, to introduce the functions $b_{1+}(t)$ and $b_{2+}(t)$ so that

$$b_{11}(t)=[b_{+1}(t)]^2,$$
$$b_{12}(t)=[b_{1+}(t)][b_{2+}(t)],$$
$$b_{22}(t)=[b_{2+}(t)]^2,$$

which when inserted into Eq. (2.7) yields the final form of the Regge amplitudes for NN scattering. These are given in Tables II, III, IV in their exact form. The leading terms in the series, valid for $s\gg4m^2-t$, are to be found in Tables V, VI, and VII.

It has recently been shown[15] that the Regge analysis of scattering problems involving spin may be decisively simplified if helicity amplitudes are introduced. To make use of these results (as we shall in Secs. IV and V), we wish to express the helicity amplitudes explicitly in terms of our factored residues b_{1+} and b_{2+}. This is most easily accomplished by using first Eqs. (4.17a–e) of GGMW to relate the helicity amplitudes $\{\phi_1,\phi_2,\phi_3,\phi_4,\phi_5\}$ to the $\{F_1,\cdots,F_5\}$, and then the inverse of our Eq. (2.2) to relate them to F_S, F_T, F_A, F_V, F_P. Use of Table II then yields the desired results, which are the following: ($T=8\pi s^{1/2}\phi$, and we use m_N as the energy unit)

$$\begin{pmatrix} T_1\\ T_2\\ T_3\\ T_4\\ T_5/2[-st(s+t-4)]^{1/2}\end{pmatrix}=2s_0\zeta/(s-4)4$$
$$\times\begin{pmatrix} 4(s+t-4) & -2ts(s+t-4) & \tfrac14\dot{Z}_\alpha t\{s(st+2t-8)+2(t-4)^2\}-\tfrac14 st^2Y_\alpha\\ st & -2ts(s+t-4) & \tfrac12\dot{Z}_\alpha t(2s+t-4)(s+t-4)-tY_\alpha(s+t-4)\\ 4(s+t-4) & -2ts(s+t-4) & \tfrac14\dot{Z}_\alpha t(st-2t+8)(s+t-4)-\tfrac14 st^2Y_\alpha\\ -st & 2ts(s+t-4) & -\tfrac12\dot{Z}_\alpha t(2s+t-4)(s+t-4)+t(s+t-4)Y_\alpha\\ 1 & -\tfrac14[st+4s+4t-16] & \tfrac18\dot{Z}_\alpha(2s+t-4)t-\tfrac14 tY_\alpha\end{pmatrix}\begin{pmatrix} b_{1+}{}^2Z_\alpha\\ b_{1+}b_{2+}\dot{Z}_\alpha\\ b_{2+}{}^2\end{pmatrix}.\quad(2.11)$$

TABLE II. Regge amplitudes for a pole in the t channel with quantum numbers $\tau P=+$, $(-)^I GP=+$. In the tables, $\zeta=(1+\tau e^{-i\pi\alpha})/2\sin\pi\alpha$.

$$F_S=\zeta\frac{2s_0}{(4m^2)^2}\left\{4m^2Z_\alpha(s,t)[b_{1+}(t)]^2+t\left[\frac{(s+\frac{1}{2}(t-4m^2))}{s_0}Z_\alpha'-\frac{\alpha(\frac{1}{2}t-2m^2+s)^2}{s(s+t-4m^2)}\left(\frac{2[(t-4m^2)/2s_0]^2}{2\alpha-1}Z_{\alpha-1}'-(\alpha-1)Z_\alpha\right)\right][b_{2+}(t)]^2\right.$$
$$\left.-(t+4m^2)\left(\frac{s+\frac{1}{2}(t-4m^2)}{s_0}\right)Z_\alpha'[b_{1+}(t)b_{2+}(t)]\right\}$$

$$F_T=-\zeta\frac{2s_0}{(4m^2)^2}\left\{t\left(\frac{t-4m^2}{2}\right)\left[\frac{Z_\alpha'}{s_0}-\frac{\alpha(s+\frac{1}{2}(t-4m^2))}{s(s+t-4m^2)}\left[\frac{2}{2\alpha-1}\left(\frac{t-4m^2}{2s_0}\right)^2Z_{\alpha-1}'-(\alpha-1)Z_\alpha\right]\right][b_{2+}(t)]^2-t\left(\frac{t-4m^2}{2s_0}\right)Z_\alpha'b_{1+}(t)b_{2+}(t)\right\}$$

$$F_A=\zeta\frac{2s_0}{(4m^2)^2}\frac{\alpha t(t-4m^2)^2}{4s(s+t-4m^2)}\left[\frac{2}{2\alpha-1}\left(\frac{t-4m^2}{2s_0}\right)^2Z_{\alpha-1}'-(\alpha-1)Z_\alpha\right][b_{2+}(t)]^2$$

$$F_V=-\zeta\frac{2s_0}{(4m^2)^2}\left\{\left[-\frac{Z_\alpha'}{s_0}+\frac{\alpha(s+\frac{1}{2}(t-4m^2))}{s(s+t-4m^2)}\left[\frac{2}{2\alpha-1}\left(\frac{t-4m^2}{2s_0}\right)^2Z_{\alpha-1}'-(\alpha-1)Z_\alpha\right]\right]t\left(\frac{t-4m^2}{2}\right)[b_{2+}(t)]^2+4m^2\left(\frac{t-4m^2}{2s_0}\right)Z_\alpha'b_{1+}(t)b_{2+}(t)\right\}$$

$$F_P=\zeta\frac{2s_0}{(4m^2)^2}\left\{(t-4m^2)\left[\frac{(s+\frac{1}{2}(t-4m^2))}{s_0}Z_\alpha'-\frac{\alpha[m^2(t-4m^2)+(s+\frac{1}{2}(t-4m^2))^2]}{s(s+t-4m^2)}\left[\frac{2}{2\alpha-1}\left(\frac{t-4m^2}{2s_0}\right)^2Z_{\alpha-1}'-(\alpha-1)Z_\alpha\right]\right][b_{2+}(t)]^2\right.$$
$$\left.-\frac{(t-4m^2)}{2s_0}(2s+t-4m^2)Z_\alpha'b_{1+}(t)b_{2+}(t)\right\}$$

We have introduced the following abbreviations:

$$\dot{Z}=dZ/ds=Z'/s_0\,,$$
$$\zeta=(1+\tau e^{-i\pi\alpha})/2\sin\pi\alpha\,,$$
$$Y_\alpha=\alpha\{[2/(2\alpha-1)][(t-4)/2s_0]^2Z_{\alpha-1}'-(\alpha-1)Z_\alpha\}\,.$$

In the asymptotic limit $s\to\infty$, $t\lesssim 0$ the relation between the helicity amplitudes and b_{1+}, b_{2+} simplifies to

$$\begin{pmatrix}T_1\\T_2\\T_3\\T_4\\T_5\end{pmatrix}=(s_0\zeta/2)(s/s_0)^\alpha\begin{pmatrix}4 & -t & t\\ t & -t & 4\\ 4 & -t & t\\ -t & t & -4\\ -2(-t)^{1/2} & (-t)^{1/2}(1+\frac{1}{4}t) & -2(-t)^{1/2}\end{pmatrix}\begin{pmatrix}b_{1+}^2\\ 2\alpha b_{1+}b_{2+}\\ \frac{1}{4}t\alpha^2 b_{2+}^2\end{pmatrix},\qquad(2.12)$$

and a simple relation between the helicity amplitudes is revealed,

$$T_1\to T_3\,,\quad T_2=-T_4\,.\qquad(2.13)$$

B. A Symmetry of *NN* Scattering

The amplitudes which describe nucleon-nucleon scattering in two different channels are related, in the graphical description of these processes, by a reversal of the external baryon lines. For example, under line reversal the amplitude describing the reaction

$$N_1+N_2\rightleftharpoons N_1'+N_2'\,,\qquad(2.11\text{a})$$

TABLE III. Regge amplitudes for a pole in the t channel with quantum numbers $\tau P=-$, $(-)^I GP=-$.

$$F_S=F_T=F_A=F_V=0$$
$$F_P=-\zeta(2s_0/4m^2)Z_\alpha b_0(t)$$

is carried into the amplitude for the reaction

$$N_1+\bar{N}_1'\rightleftharpoons N_2'+\bar{N}_2\,,\qquad(2.11\text{b})$$

TABLE IV. Regge amplitudes for a pole in the t channel with quantum numbers $\tau P=-$, $(-)^I GP=+$.

$$F_S=-\zeta\frac{t}{4m^2}\frac{s_0}{s}\frac{\alpha(s+\frac{1}{2}(t-4m^2))}{(s+t-4m^2)}$$
$$\times\left[\frac{2}{2\alpha-1}\left(\frac{t-4m^2}{2s_0}\right)^2Z_{\alpha-1}'-(\alpha-1)Z_\alpha\right]b_1(t)$$
$$F_T=\frac{-(t-4m^2)}{2s+t-4m^2}F_S$$
$$F_A=-\zeta(t/4m^2)Z_\alpha'b_1(t)-F_S$$
$$F_V=-F_T$$
$$F_P=\zeta Z_\alpha'b_1(t)+F_S$$

TABLE V. Leading terms in the expansion of the Regge amplitude for a pole in the t channel with quantum numbers $\tau P=+$, $(-)^I GP=+$.

$$F_S \to \zeta\frac{2s_0}{(4m^2)^2}\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha}\left\{4m^2[b_{1+}(t)]^2\left[1-\frac{\alpha(\alpha-1)}{2(2\alpha-1)}x^2\right]-\alpha(t+4m^2)\right.$$
$$\left.\times[b_{1+}(t)b_{2+}(t)]\left[1-\frac{(\alpha-1)(\alpha-2)}{2(2\alpha-1)}x^2\right]+\alpha t[b_{2+}(t)]^2\left[\alpha-\frac{(\alpha-1)(\alpha-2)^2x^2}{2(2\alpha-1)}\right]\right\}$$

$$F_T \to \zeta\frac{2s_0}{(4m^2)^2}\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha} t\alpha x[b_{1+}(t)-\alpha b_{2+}(t)]b_{2+}(t)$$

$$F_A \to -\zeta\frac{2s_0}{(4m^2)^2}\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha} t\alpha(\alpha-1)x^2[b_{2+}(t)]^2$$

$$F_V \to \zeta\frac{2s_0}{(4m^2)^2}\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha} \alpha x[\alpha t b_{2+}(t)-4m^2 b_{1+}(t)]b_{2+}(t)$$

$$F_P \to \zeta\frac{2s_0}{(4m^2)^2}\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha} \alpha(t-4m^2)b_{2+}(t)\left\{-b_{1+}(t)\left[1-\frac{(\alpha-1)(\alpha-2)}{2(2\alpha-1)}x^2\right]+b_{2+}(t)\left[\alpha-(\alpha-1)x^2\left(\frac{(\alpha-2)^2}{2(2\alpha-1)}+\frac{4m^2}{4m^2-t}\right)\right]\right\}$$

in which $x\to -x/(1+2x)$, $s\to -s(1+2x)$. The amplitudes for these two processes are related,[5] in the limit of high energies and low-momentum transfers, by a multiplicative factor $\tau\epsilon$. They, therefore, satisfy a "generalized Pomeranchuk relation." τ is the signature,[1] or orbital parity of the Regge trajectory, and in NN scattering $\epsilon=(-)^I G\tau$. The resultant factor is therefore $(-)^I G$ which, for the neutral member of an isotopic multiplet, is the charge conjugation quantum number C.

Examples of this symmetry are provided by the following reactions in which the ρ meson is exchanged:

Pole	Asymptotic amplitudes	
ρ	$T(\pi^0+n\to\pi^-+p)$	(2.13a)
	$-T(\pi^++n\to\pi^0+p)$	(2.13b)
	$-T(\pi^0+\bar{p}\to\pi^-+\bar{n})$	(2.13c)
	$T(\pi^++\bar{p}\to\pi^0+\bar{n})$	(2.13d)
ρ	$T(p+n\to p+n)$	(2.14a)
	$-T(\bar{n}+n\to p+\bar{p})$	(2.14b)
	$-T(p+\bar{p}\to\bar{n}+n)$	(2.14c)
	$T(\bar{n}+\bar{p}\to\bar{n}+\bar{p})$.	(2.14d)

TABLE VI. Leading terms in the expansion of the Regge amplitudes for a pole in the t channel with quantum numbers $\tau P=-$, $(-)^I GP=-$.

$$F_S=F_T=F_A=F_V=0$$

$$F_P \to -\zeta\frac{2s_0}{4m^2}\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha} b_0(t)\left[1-\frac{\alpha(\alpha-1)x^2}{2(2\alpha-1)}\right]$$

III. ASYMPTOTIC AMPLITUDES DUE TO THE EXCHANGE OF P, ω, ρ, AND π MESONS

In this section, we construct the contributions to the invariant amplitudes describing NN scattering arising from the exchange of the P, ω, ρ, and π mesons. We can then compare the asymptotic forms of these expressions to those given by the Regge theory applied to the corresponding trajectories, and identify the residues b_i with appropriate coupling constants by comparing the amplitudes[1] at $t=m_\rho^2$, m_π^2, $\cdots$.

We shall first consider the Pomeranchuk trajectory, having the quantum numbers of the vacuum and $\alpha_P(0)=1$. It is possible that there is a spin-2^+ resonance occurring on this trajectory[21] at $t\sim 1$ $(\text{GeV})^2$. We may

TABLE VII. Leading terms in the expansion of the Regge amplitudes for a pole in the t channel with quantum numbers $\tau P=-$, $(-)^I GP=+$.

$$F_S \to \zeta\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha-1}\alpha(\alpha-1)\frac{t}{4m^2}b_1(t)$$

$$F_T \to -\zeta\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha-1}\alpha(\alpha-1)x^2\frac{t}{4m^2}b_1(t)$$

$$F_A \to -\zeta\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha-1}\alpha^2\frac{t}{4m^2}b_1(t)$$

$$F_V=-F_T$$

$$F_P \to -\zeta\left(\frac{2s+t-4m^2}{2s_0}\right)^{\alpha-1}\alpha\left[(\alpha-1)\frac{t}{4m^2}+1\right]b_1(t)$$

[21] For preliminary experimental evidence for such a resonance, see: W. Selove, V. Hogopian, H. Brody, A. Baker, and E. Leboy, Phys. Rev. Letters **9**, 272 (1962); J. J. Veillet, J. Hennessy, H. Bingham, M. Bloch, D. Drijand, A. Lagarrique, P. Mittner, A. Rousset, G. Bellini, M. di Corato, E. Fiorini, and P. Negri, *ibid.* **10**, 29 (1963).

identify the Pomeranchuk pole residues with the coupling constants of this spin-2 resonance to the nucleon. To do this, we must first construct the contribution of a spin-2 meson, C, to the invariant amplitudes describing NN scattering.

The propagator for a spin-2 meson must be a tensor of rank four. Its most general form is, therefore,

$$\begin{aligned} D_{\mu\nu\lambda\sigma}(q^2)=a\{&\delta_{\mu\lambda}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\lambda}+B\delta_{\mu\nu}\delta_{\lambda\sigma}\\ &+C(q_\lambda q_\sigma\delta_{\mu\nu}+q_\mu q_\nu\delta_{\lambda\sigma})/m^2\\ &+D(q_\nu q_\sigma\delta_{\mu\lambda}+q_\mu q_\lambda\delta_{\nu\sigma}+q_\nu q_\lambda\delta_{\mu\sigma}+q_\mu q_\sigma\delta_{\nu\lambda})/m^2\\ &+E(q_\mu q_\nu q_\lambda q_\sigma)/m^4\}(q^2+m^2)^{-1}, \end{aligned}\quad (3.1)$$

where we have taken into account the symmetries $D_{\mu\nu\lambda\sigma}=D_{\nu\mu\lambda\sigma}$ and $D_{\mu\nu\lambda\sigma}=D_{\lambda\sigma\mu\nu}$. At the pole $q^2=-m^2$, the propagator is divergenceless, $q_\mu D_{\mu\nu\lambda\sigma}=0$, and traceless, $D_{\mu\mu\lambda\sigma}=0$. From these two conditions we find $B=C=-\frac{2}{3}$, $D=1$, and $E=\frac{4}{3}$. The factor a is determined to have the value $\frac{1}{2}$, so that if a polarization tensor $\epsilon_{\mu\nu}$ of the meson is normalized to 1, then $\epsilon_{\mu\nu}D_{\mu\nu\lambda\sigma}\epsilon_{\lambda\sigma}=1$.

In the Born approximation, the coupling of the C meson to two nucleons takes the form

$$(1/4m_N^2)(\chi_{CNN}-\xi_{CNN})\Sigma_\mu\Sigma_\nu+(i/4m_N)\xi_{CNN}[\Sigma_\mu\gamma_\nu+\Sigma_\nu\gamma_\mu],\quad (3.2)$$

where $\Sigma_\mu=(p+p')_\mu$.

From these results we readily see that the C-meson pole term in the amplitude has the form

$$\begin{aligned} T=&\frac{1}{t-m_C^2}\bar{u}_2'\left[\frac{(\chi_{CNN}-\xi_{CNN})}{4m_N^2}\Sigma_\mu'\Sigma_\nu'+\frac{i\xi_{CNN}}{4m_N}(\Sigma_\mu'\gamma_\nu+\Sigma_\nu'\gamma_\mu)\right]u_2\\ &\times\bar{u}_1'\left[\frac{(\chi_{CNN}-\xi_{CNN})}{4m_N^2}\Sigma_\mu\Sigma_\nu+\frac{i\xi_{CNN}}{4m_N}(\Sigma_\mu\gamma_\nu+\Sigma_\nu\gamma_\mu)\right]u_1-\frac{1}{3}\frac{1}{t-m_C^2}\bar{u}_2'u_2\bar{u}_1'u_1\\ &\times\left[\chi_{CNN}\left(\frac{t}{4m_N^2}-1\right)-\xi_{CNN}\frac{t}{4m_N^2}\right]^2, \end{aligned}\quad (3.2)$$

where $\Sigma_\mu'=(p_2+p_2')_\mu$, $\Sigma_\mu=(p_1+p_1')_\mu$.

Using Eq. (2.21) of ALV, which states that

$$2im_N[\bar{u}_1'\Sigma_\mu'\gamma_\mu u_1\bar{u}_2'u_2+\bar{u}_1'u_1\bar{u}_2'\Sigma_\mu\gamma_\mu u_2]=(4m_N^2-t-2s)(S+P)-4m_N^2V+tT\quad (3.3a)$$

and

$$\bar{u}_1'\Sigma_\mu'\gamma_\mu u_1\bar{u}_2'\Sigma_\nu\gamma_\nu u_2=-(2s+t-4m_N^2)V+tA-4m_N^2P,\quad (3.3b)$$

we find that the C-pole terms in the invariant amplitudes are:

$$(4m_N^2)^2(t-m_C^2)F_S=\chi_{CNN}(\chi_{CNN}-\xi_{CNN})(4m_N^2-t-2s)^2-\tfrac{1}{3}[\chi_{CNN}(t-4m_N^2)-t\xi_{CNN}]^2,\quad (3.4a)$$

$$(4m_N^2)^2(t-m_C^2)F_T=(\chi_{CNN}-\xi_{CNN})\xi_{CNN}t(4m_N^2-t-2s),\quad (3.4b)$$

$$(4m_N^2)^2(t-m_C^2)F_A=-2\xi_{CNN}^2m_N^2t,\quad (3.4c)$$

$$(4m_N^2)^2(t-m_C^2)F_V=-4\chi_{CNN}\xi_{CNN}m_N^2(4m_N^2-t-2s),\quad (3.4d)$$

$$(4m_N^2)^2(t-m_C^2)F_P=8\xi_{CNN}^2m_N^4+(\chi_{CNN}-\xi_{CNN})\times\xi_{CNN}(4m_N^2-t-2s)^2.\quad (3.4e)$$

These expressions may be compared to those in Table VI. In particular, we can, at $t=m_C^2$, identify the Pomeranchuk Regge pole parameters α, b_{1+}, and b_{2+} with various properties of the C meson. At the position of the resonance, $t=m_C^2$, we must have $\mathrm{Re}\,\alpha_P(m_C^2)=2$. Also $\mathrm{Im}\,\alpha_P(m_C^2)\equiv I_P$ is related to the width,[1]

$$m_C\Gamma_C=I_P/\epsilon_C,\quad (3.5)$$

where $\epsilon_C=\mathrm{Re}[d\alpha_P(t)/dt|_{t=m_C^2}]$, and we find

$$\begin{aligned} b_{1+}^P(m_C^2)/[\pi\epsilon_Cs_0]^{1/2}&=[(\chi_{CNN}-\xi_{CNN})+\xi_{CNN}m_C^2/(4m_N^2-m_C^2)]/m_N,\\ b_{2+}^P(m_C^2)/[\pi\epsilon_Cs_0]^{1/2}&=2m_N\xi_{CNN}/(4m_N^2-m_C^2). \end{aligned}\quad (3.6)$$

In the Appendix we discuss the decay rate and branching ratio of the C meson.

In a similar way, we may compare the Feynman amplitude corresponding to ρ exchange with the associated Regge pole contribution, to identify the residues $b_i^\rho(t)$ at $t=m_\rho^2$. For the ρ-pole term in the amplitude, we have

$$T(s,t)=-\gamma_{NN\rho}^2\bar{u}_1'\left[\gamma_\mu-\left(\frac{\mu_{NN\rho}}{2m_N}\right)\sigma_{\mu\nu}(p_1'-p_1)_\nu\right]\tau_\alpha u_1\times\frac{\bar{u}_2'[\gamma_\mu-(\mu_{NN\rho}/2m_N)\sigma_{\mu\nu}(p_2'-p_2)_\nu]\tau_\alpha u_2}{t-m_\rho^2},\quad (3.7)$$

which can be reduced to the form

$$\begin{aligned} T(s,t)=&\frac{1}{2}\begin{pmatrix}-3\\1\end{pmatrix}\left(\frac{-2\gamma_{NN\rho}^2}{t-m_\rho^2}\right)\\ &\times\left\{S[4m_N^2-2s-t]\frac{\mu_{\rho NN}}{4m_N^2}+P\left[1-\frac{2s+t}{4m_N^2}\right]\mu_{\rho NN}(1+\mu_{\rho NN})\right.\\ &\left.+\frac{\mu_{\rho NN}(1+\mu_{\rho NN})}{4m_N^2}tT+(1+\mu_{\rho NN})V\right\}, \end{aligned}\quad (3.8)$$

using the relations

$$\begin{aligned} -\bar{u}_{p'}\sigma_{\mu\nu}(p'-p)_\nu u_p&=\bar{u}_{p'}[2m_N\gamma_\mu+i(p'+p)_\mu]u_p,\\ (p_1+p_1')_\mu(p_2+p_2')_\mu&=u-s=4m_N^2-t-2s, \end{aligned}\quad (3.9)$$

and Eq. (3.3b). In the column vector, the first row refers to $I=0$ in the s channel, and the second to $I=1$.

Near the ρ-meson pole, therefore, we have (dropping isospin factors)

$$F_S=\left(\frac{-2\gamma_{NN\rho}^2}{t-m_\rho^2}\right)\frac{(4m_N^2-t-2s)}{4m_N^2}\mu_{\rho NN}\,, \qquad (3.10a)$$

$$F_T=\left(\frac{-2\gamma_{NN\rho}^2}{t-m_\rho^2}\right)\frac{t}{4m_N^2}\mu_{\rho NN}(1+\mu_{\rho NN})\,, \qquad (3.10b)$$

F_A is not singular, (3.10c)

$$F_V=\left(\frac{-2\gamma_{NN\rho}^2}{t-m_\rho^2}\right)(1+\mu_{\rho NN})\,, \qquad (3.10d)$$

$$F_P=\left(\frac{-2\gamma_{NN\rho}^2}{t-m_\rho^2}\right)\frac{(4m_N^2-t-2s)}{4m_N^2}\mu_{\rho NN}(1+\mu_{\rho NN})\,. \qquad (3.10e)$$

We compare these results to those arising from the ρ trajectory. At $t=m_\rho^2$, $\mathrm{Re}\alpha_\rho(m_\rho^2)=1$, and as before, we have

$$I_\rho=m_\rho\Gamma_\rho\epsilon_\rho\,,$$

where $\alpha_\rho(m_\rho^2)=1+iI_\rho$ and $\epsilon_\rho=\mathrm{Re}[d\alpha_\rho(t)/dt|_{t=m_\rho^2}]$. We then find that the $b_i^\rho(t)$ are related, at $t=m_\rho^2$, to the coupling constants $\gamma_{\rho NN}$ and $\mu_{\rho NN}$ as follows:

$$[1-(m_\rho^2/4m_N^2)]b_{1+}^\rho(m_\rho^2)[\pi\tfrac{1}{2}\epsilon_\rho]^{-1/2} =2\gamma_{\rho NN}[1+\mu_{\rho NN}m_\rho^2/4m_N^2]\,, \qquad (3.11a)$$

$$[1-(m_\rho^2/4m_N^2)]b_{2+}^\rho(m_\rho^2)[\pi\tfrac{1}{2}\epsilon_\rho]^{-1/2} =2\gamma_{\rho NN}(1+\mu_{\rho NN})\,. \qquad (3.11b)$$

The corresponding formulas for the ω Regge pole are exactly analogous to those of the ρ, since the only difference is that of isospin, which we take care of with the matrix Λ, (see Eq. 2.0e).

Of those Regge poles associated with meson systems having zero spin, the most prominent contributor to the NN-scattering amplitude is likely to be that corresponding to the pion, since $\mathrm{Re}\alpha_\pi(t)$ is zero for the lowest t, $\alpha_\pi(m_\pi^2)=0$. This trajectory has $I=1$, $C=+$, $\tau=+$, $\tau P=-$. Near $t=m_\pi^2$,

$$T(s,t)=-g_{NN\pi}^2P/(t-m_\pi^2)\begin{pmatrix}-3\\1\end{pmatrix},$$

and, therefore,

$$b_0^\pi(m_\pi^2)/\pi\epsilon_\pi=2g_{NN\pi}^2/(2s_0)^{1/2}.$$

In considering various trajectories which may contribute to NN scattering, we should like to mention briefly some recent speculations on the existence of another trajectory with $C=+$, for which $\alpha(0)$ lies in the region 0 to 1. Igi[14] has shown that the data on π^+p and π^-p scattering require some singularity in the J plane which lies in the region 0 to 1 for forward scattering. Let this singularity, which has $C=+$ and $\tau=+$, since it is coupled to the two-pion system, be labeled P'. As we shall see in Sec. V, and as suggested on the basis of a spinless treatment of NN scattering by Hadjioannou *et al.*,[13] such a singularity is also needed to cancel the contribution of the ω Regge pole in NN scattering. It must, therefore, have $I=0$, and is a companion to the Pomeranchuk trajectory in that they both have the quantum numbers of the vacuum. Igi has suggested[14] that the P' be associated with the ABC anomaly,[22–24] but this seems inappropriate because the trajectory associated with the ABC anomaly must have $\alpha\approx0$ near $t=0$. If the P' singularity is a pole, rather than a branch cut, there exists the possibility of associating P' with a resonance with $J=2$ in the region $t>4m_\pi^2$. We would look for such a resonance in the 1 to 1.5 GeV region, which still remains virtually unexplored.[25] However, the P' trajectory may not reach the line $\mathrm{Re}\alpha=2$, or even if it does, $\mathrm{Im}\alpha$ may be large, so that a resonance would not occur.

Mandelstam[26] has recently investigated the contribution of a class of multiparticle intermediate states to the partial-wave amplitude. He concludes that they give rise to cuts in the angular-momentum plane which are in general present up to $t=0$. If this conclusion is correct, it appears to us much more plausible to regard the P' singularity as the cut associated with the P pole, rather than as a second vacuum trajectory. For further comments, see Sec. V of this paper.

IV. CROSS SECTIONS FOR NUCLEON-NUCLEON SCATTERING

A. Elastic Differential Cross Section

In terms of the couplings η_i and ϕ_i of the ith Regge trajectory to pairs of incident or outgoing nucleons in the states without helicity flip (η) and with helicity flip (ϕ), the elastic differential cross section takes the form[15]

$$d\sigma/dt=\frac{1}{4\pi}\sum_{ij}\mathrm{Re}(\zeta_i{}^*\zeta_j)\left(\frac{s}{s_0{}^i}\right)^{\alpha_i-1} \times\left(\frac{s}{s_0{}^j}\right)^{\alpha_j-1}(\eta_N{}^i\eta_N{}^j+\phi_N{}^i\phi_N{}^j)^2\,, \qquad (4.1)$$

where

$$\eta=b_{1+}-(\alpha t/4m^2)b_{2+}$$

and

$$\phi=(-t/4m^2)^{1/2}(b_{1+}-\alpha b_{2+}).$$

[22] N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters **7**, 35 (1961).
[23] J. Button, G. R. Kalbfleisch, G. R. Lynch, B. C. Maglic, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. **126**, 1858 (1962).
[24] B. Richter, Phys. Rev. Letters **9**, 217 (1962).
[25] See, however, Ref. 21.
[26] S. Mandelstam, lecture given at the California Institute of Technology (unpublished) and private communication.

Using Eq. (2.12) we may easily construct the contribution to the differential cross sections for NN and $N\bar{N}$ scattering from the P, P', ω, and ρ trajectories. For pp scattering, we find

$$16\pi s(s-4m_N^2)\frac{d\sigma_{pp}}{dt}=D_{PP}\left[\frac{2s+t-4m_N^2}{2s_0^P}\right]^{2\alpha_P(t)}+2D_{P\omega}\left[\frac{2s+t-4m_N^2}{2s_0^P}\right]^{\alpha_P(t)}\left[\frac{2s+t-4m_N^2}{2s_0^\omega}\right]^{\alpha_\omega(t)}$$
$$+2D_{PP'}\left[\frac{2s+t-4m_N^2}{2s_0^P}\right]^{\alpha_P(t)}\left[\frac{2s+t-4m_N^2}{2s_0^{P'}}\right]^{\alpha_{P'}(t)}+D_{\omega\omega}\left[\frac{2s+t-4m_N^2}{2s_0^\omega}\right]^{2\alpha_\omega(t)}$$
$$+2D_{\omega P'}\left[\frac{2s+t-4m_N^2}{2s_0^\omega}\right]^{\alpha_\omega(t)}\left[\frac{2s+t-4m_N^2}{2s_0^{P'}}\right]^{\alpha_{P'}(t)}+D_{P'P'}\left[\frac{2s+t-4m_N^2}{2s_0^{P'}}\right]^{2\alpha_{P'}(t)}. \quad (4.2)$$

The result for $p\bar{p}$ scattering is the same except for a minus sign on the terms $D_{P\omega}$ and $D_{P'\omega}$. Since no $I=1$ pole is included, $d\sigma_{np}/dt\approx d\sigma_{pp}/dt$.

The coefficients D_{ij} are found to have remarkably simple expressions,

$$\sin^2\frac{\pi\alpha_P(t)}{2}D_{PP}(t)=\left[(b_{1+}{}^P)^2-\alpha_P{}^2\frac{t(b_{2+}{}^P)^2}{4m_N{}^2}\right]^2(s_0{}^P)^2\left(1-\frac{t}{4m_N{}^2}\right)^2, \quad (4.3)$$

and

$$\frac{\sin\pi\alpha_P\sin\pi\alpha_\omega D_{P\omega}}{[1+\tau_P\cos\pi\alpha_P+\tau_\omega\cos\pi\alpha_\omega+\tau_P\tau_\omega\cos\pi(\alpha_P-\alpha_\omega)]}=\left[b_{1+}{}^Pb_{1+}{}^\omega-\frac{t}{4m_N{}^2}\alpha_P\alpha_\omega b_{2+}{}^Pb_{2+}{}^\omega\right]^2 s_0{}^Ps_0{}^\omega\left(1-\frac{t}{4m_N{}^2}\right)^2, \quad (4.4)$$

where τ_P, τ_ω indicate the signature of the P, ω trajectory. All the other D functions can be obtained simply by changing the indexes.

The circumstance that the coefficients D_{ij} are perfect squares is a result of the facts that the amplitudes can be factored and that all particles are nucleons.

These same results have also been obtained directly by expressing the cross section in terms of the Fermi amplitudes F_I ($I=S,T,A,V,P$) and evaluating the relevant traces.[27]

B. Polarized Cross Sections

The helicity representation also allows a simple discussion of polarization phenomena.[15] In the scattering of unpolarized particles, it is possible to polarize the particles normal to the scattering plane. If the fraction of the scattered particles with spin up minus the fraction of particles with spin down is called P, when the particles have scattered at a given angle to the left, P is given asymptotically by

$$P\frac{d\sigma}{dt}=\frac{1}{2\pi}\sum_{ij}\text{Im}(\zeta_i{}^*\zeta_j)\left(\frac{s}{s_0{}^i}\right)^{\alpha_i-1}$$
$$\times\left(\frac{s}{s_0{}^j}\right)^{\alpha_j-1}(\eta_N{}^i\eta_N{}^j+\phi_N{}^i\phi_N{}^j)\phi_N{}^i\eta_N{}^j. \quad (4.5)$$

As before, use of Eq. (2.12) allows us to evaluate Eq. (4.5) explicitly. We find, asymptotically (including only $I=0$ poles),

$$2\pi P\frac{d\sigma}{dt}=L_{PP'}\left(\frac{s}{s_0{}^P}\right)^{\alpha_P-1}\left(\frac{s}{s_0{}^{P'}}\right)^{\alpha_{P'}-1}$$
$$+L_{P\omega}\left(\frac{s}{s_0{}^P}\right)^{\alpha_P-1}\left(\frac{s}{s_0{}^\omega}\right)^{\alpha_\omega-1}$$
$$+L_{P'\omega}\left(\frac{s}{s_0{}^{P'}}\right)^{\alpha_{P'}-1}\left(\frac{s}{s_0{}^\omega}\right)^{\alpha_\omega-1}. \quad (4.6)$$

The coefficients L_{ij} are again simple and have the form,

$$\frac{4\sin\pi\alpha_P\sin\pi\alpha_{P'}L_{PP'}}{\tau_P\sin\pi\alpha_P-\tau_{P'}\sin\pi\alpha_{P'}+\tau_P\tau_{P'}\sin\pi(\alpha_P-\alpha_{P'})}$$
$$=(b_{1+}{}^Pb_{1+}{}^{P'}+\alpha_P\alpha_{P'}(-t/4m_N{}^2)b_{2+}{}^Pb_{2+}{}^{P'})$$
$$\times(\alpha_Pb_{2+}{}^Pb_{1+}{}^{P'}-\alpha_{P'}b_{2+}{}^{P'}b_{1+}{}^P)$$
$$\times\left(1-\frac{t}{4m_N{}^2}\right)^2\frac{(-t)^{1/2}}{2m_N}. \quad (4.7)$$

The other coefficients are obtained by changing the labels appropriately. That polarization phenomena occur only as a result of the interference between various trajectories can be seen from Eq. (4.5).

C. Total Cross Sections

By the optical theorem, the total cross section is related to the imaginary part of the forward-scattering

[27] D. H. Sharp, Ph.D. thesis to be submitted to California Institute of Technology (unpublished).

amplitude,

$$\sigma_{\text{tot}}(s)=\frac{-1}{[s(s-4m_N^2)]^{1/2}}\,\text{Im}T(s,t=0). \quad (4.8)$$

To apply this formula, we need to evaluate the spin average of each of the Fermi invariants in the forward direction. We shall do this by computing the two helicity amplitudes $T(++,++)$ and $T(+-,-+)$ for $t=0$, where the $\pm$ signs denote the helicities of particles (1′, 2′, 2, 1). We find

	$(++,++)$	$(+-,-+)$
S	$4m_N^2$	$4m_N^2$
T	$-4m_N^2$	$4m_N^2$
A	$-(2s-4m_N^2)$	$(2s-4m_N^2)$
V	$(2s-4m_N^2)$	$(2s-4m_N^2)$
P	0	$0.$

(4.9)

Since a trajectory with quantum numbers $\tau P=-$, $(-)^I GP=-$ gives a contribution only to F_P, it makes no contribution to the total cross section. In particular, there will be no terms in formulas for the total cross sections arising from the π-meson and η-meson trajectories. The contributions to the spin-averaged, total cross sections from the P, P', ω, and ρ trajectories are

$$\sigma_{pp}=\{B^P-B^\omega R_\omega(v)+B^{P'}R_{P'}(v)-B^\rho R_\rho(v)\}\times(1-1/v^2)^{-1/2}, \quad (4.10a)$$

$$\sigma_{np}=\{B^P-B^\omega R_\omega(v)+B^{P'}R_{P'}(v)+B^\rho R_\rho(v)\}\times(1-1/v^2)^{-1/2}, \quad (4.10b)$$

$$\sigma_{p\bar{p}}=\{B^P+B^\omega R_\omega(v)+B^{P'}R_{P'}(v)+B^\rho R_\rho(v)\}\times(1-1/v^2)^{-1/2}, \quad (4.10c)$$

where

$$B=(2m_N^2/s_0)^{[\alpha(0)-1]}[b_{1+}(0)]^2,$$

$$v=(s/2m_N^2)-1,$$

$$R(v)=\frac{\sqrt{\pi}[\alpha(0)]!}{2^{\alpha(0)}[\alpha(0)-\frac{1}{2}]!v}P_{\alpha(0)}(v).$$

V. AN ANALYSIS OF RECENT DATA ON NN AND $N\bar{N}$ SCATTERING

We have analyzed the data reported by Diddens *et al.*[2] on the total cross sections for pp and np scattering and those of Lindenbaum *et al.*[11] on the $p\bar{p}$ cross sections. We find that the presently available data indicate:

$$B^P\approx38\text{ mb},\quad B^{P'}\approx53\text{ mb},\quad B^\omega\approx48\text{ mb},\quad B^\rho\approx-9\text{ mb},$$
$$\alpha_{P'}\approx0.3,\qquad \alpha_\omega\approx0.3,\qquad \alpha_\rho\approx0.4. \quad (5.1)$$

We should like to make several comments on our analysis and its results:

(i) The inclusion of the nucleon's spin does not give any appreciable modification of the structure of the Regge analysis of the total cross sections.

(ii) A study of the Legendre functions, $P_{\alpha(0)}(v)$, indicates that for $\alpha<2$, P_α is represented by its leading term to better than 10% for $v>2$. Since $v=(E_{\text{lab}})/m$, it is certainly sufficient, for incident energies above 2 GeV, to keep only the leading term in any practical analysis of data. Moreover, the replacement of the Legendre functions of the first kind, $P_\alpha(v)$, by Legendre functions of the second kind, $Q_{-\alpha-1}(v)$, does not alter the fact that only the first term, v^α, in the expansion of these functions need be kept in the analysis, even though the $Q_{-\alpha-1}(v)$ are singular at $v=+1$. This simplifies the analysis, but eliminates the hope that perhaps the introduction of a second vacuum trajectory with α in the range 0 to 1 could be avoided provided that one included the full contribution from the Regge poles on the Pomeranchuk, omega, rho, and "ABC" trajectories.

(iii) Our analysis requires that the location of a possible second vacuum pole, $\alpha_{P'}(0)$, be significantly larger than zero, so that it is unlikely that the trajectory could be associated with the ABC anomaly.

(iv) Our results are different from those of Hadjioannou *et al.*,[13] who arbitrarily assumed $\alpha_\omega(0)=\alpha_{P'}(0)=0.5$ and neglected the ρ trajectory.

(v) The sign of the ρ term is opposite to that of the ω term. If a pole analysis is to be taken at all seriously, this is puzzling since it should be positive. This discrepancy may well arise from present inaccuracies in the np data. Alternatively, this may mean that the cut associated with the ρ trajectory is not small near $t=0$, and, indeed, overrides the pole part of the contribution.

(vi) We can interpret our results for the P and P' trajectories as follows. The analysis of the data indicates the presence of an additional singularity besides the P, ω, and ρ poles. This we attribute to a cut associated with the P trajectory. If the cut is approximated, near $t=0$, by a pole, then this pole is described by the parameters we have associated with the P', and whose numerical values are as given above. In so doing, we have ignored possible cuts associated with the ρ and ω.

(vii) This analysis suggests a possible explanation for the apparent lack of shrinkage[28,29] in the πp diffraction peaks. Note that the pp cross sections receive contributions from the P, P', and ω trajectories. (We suppose the ρ contribution to be small.) Each of these contributions is individually large, but the contribution of the P' is cancelled out by that of the ω, leaving just the P as the dominant contributor. In πp scattering, on the other hand, the ω can not contribute at all, which leaves the P' as a competitor of the P. These two contributions could well combine to give a resultant shrinkage which is much less rapid, over a given range of s, than that observed in pp scattering. Note that this explanation does not depend in any essential way on the supposition

[28] C. C. Ting, L. W. Jones, and M. L. Perl, Phys. Rev. Letters **9**, 468 (1962).

[29] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **10**, 376 and 543 (1963).

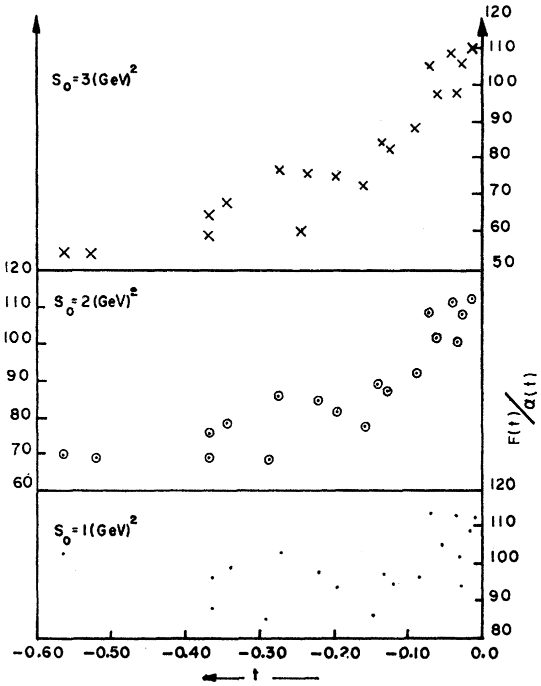

FIG. 1. $F(t)/\alpha(t)$ versus t for $s_0=1$, 2, 3 $(\text{GeV})^2$. The experimental uncertainty in each point is typically about 15%.

that the P' is a pole, rather than a cut associated with the P trajectory.

Finally, we have analyzed the data of Diddens *et al.*[3] on the pp elastic differential cross sections. These data lie in the range $12<(s/2m_N^2)-1=E_L/m_N<28$ and $0\lesssim -t<0.60$ GeV^2.

Only the Pomeranchuk contribution was included. The cross section is then given by Eqs. (4.2) and (4.3),

$$\left[\frac{d\sigma}{dt}\bigg/\frac{d\sigma}{dt}\bigg|_{t=0}\right]$$
$$=\left[\left(b_{1+}^2(t)-\alpha^2(t)b_{2+}^2(t)\frac{t}{4m_N^2}\right)\right.$$
$$\left.\times\left(1-\frac{t}{4m_N^2}\right)\frac{1}{b_1^2(0)}\right]^2\left[\frac{2s+t-4m_N^2}{2s_0}\right]^{2\alpha_P(t)-2}. \quad (5.2)$$

We note that in this one-pole approximation, the differential cross section involves only *one* unknown function, namely,

$$F(t)=b_{1+}^2(t)-\alpha^2(t)b_{2+}^2(t)(t/4m_N^2). \quad (5.3)$$

We assume for $\alpha(t)$ the linear behavior

$$\alpha(t)=1+t, \quad (5.4)$$

in accord with existing data.

According to Gell-Mann's ghost suppression mechanism,[4] the residue $F(t)$ must contain a factor of $\alpha(t)$ in order to eliminate the possibility of a ghost at $\alpha=0$ $[t\sim-1\ (\text{GeV})^2]$. The resulting quantity, $F(t)/\alpha(t)$, we expect to be nearly constant for small negative t.

The arbitrary parameter s_0 is to be chosen so that $F(t)/\alpha(t)$ varies as slowly as possible with t. We try the values $s_0=1$, 2, 3 $(\text{GeV})^2$. Results are summarized in Fig. 1. We see from the figure that the function $F(t)/\alpha(t)$ has a linear behavior for $t\lesssim -0.40$ $(\text{GeV})^2$. Beyond this point, $F(t)/\alpha(t)$ shows a marked increase reflecting a corresponding increase in the experimental value of $d\sigma/dt$. The graphs show quite clearly that the function $F(t)/\alpha(t)$ is most nearly constant for $s_0=1$ $(\text{GeV})^2$.

ACKNOWLEDGMENTS

The authors are deeply grateful to Professor Murray Gell-Mann for suggesting this problem and for many stimulating and helpful conversations. One of the authors (DHS) would also like to thank Professor A. C. Zemach for some helpful comments, and Professor David Judd of the Lawrence Radiation Laboratory, Berkeley, California, for the privilege of association with his group during the summer of 1962.

APPENDIX: DECAY RATE OF THE SPIN-2 POMERANCHUK RESONANCE[30]

The Regge approach seems to afford an explanation of the constancy of total cross sections at high energies if the existence is assumed of a Regge trajectory $\alpha_P(t)$, having the quantum numbers of the vacuum, positive signature, and $\alpha_P(0)=1$. Accepting this, it is possible that a spin-2 resonance, C, having the same quantum numbers and mass $m_C^2\sim 1$ $(\text{GeV})^2$ may lie on the Pomeranchuk trajectory.[31,32] Such a resonance should show up as a peak in $T=0$ $\pi\pi$ scattering and in $K\bar{K}$ scattering. It is the purpose of this note to discuss the two-body decay modes of this resonance.

The graviton G is coupled universally to the symmetrized stress-energy-momentum tensor $T_{\mu\nu}$. We assume, in complete analogy with Gell-Mann and Zachariasen's treatment[33] of the ρ meson, that the spin-2 resonance is a slightly unstable spin-2 meson, that it couples strongly to baryon and pion pairs, and that it dominates the gravitational form factors.[34] If we write a dispersion relation for the vertex shown in Fig. 2, assume the approximation depicted in Fig. 3, and define

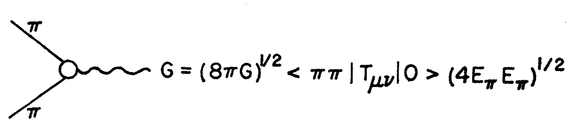

FIG. 2. Decay vertex for C meson.

[30] Murray Gell-Mann has also obtained these results independently.
[31] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **8**, 41 (1962).
[32] C. Lovelace, Nuovo Cimento **25**, 730 (1962).
[33] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).
[34] P. G. O. Freund, Phys. Letters **2**, 136 (1962).

the coupling indicated in Fig. 4, we obtain

$$(8\pi G)^{1/2}E_\pi = (8\pi G)^{1/2}(m_C{}^3/\chi_{CG})(E_\pi/m_\pi)\chi_{C\pi\pi}[1/(q^2+m_C{}^2)]|_{q^2=0} + \text{(contributions from higher mass intermediate states)}. \quad \text{(A1)}$$

Thus, $\chi_{C\pi\pi} \sim (m_\pi/m_C)\chi_{CG}$. Replacing the pions by any other pair of external particles to which the C is strongly coupled gives the same relation, with the mass of the new particles replacing the pion mass. This is the principle of universality, which would hold rigorously if the C meson were massless.

We shall now compute the decay rate for $C \rightarrow 2\pi$, $2K$. The amplitude for the decay $C \rightarrow \pi^+, \pi^-$ is

$$T = (\chi_{C\pi\pi}/2m_\pi)\epsilon_{\mu\nu}(p-p')_\mu(p-p')_\nu\,, \quad \text{(A2)}$$

where p, p' are the momenta of the pions and $\epsilon_{\mu\nu}$ is the polarization tensor of the C meson, which is at rest in the c.m. system. With inclusions of isotopic spin factors, we find for the decay rate

$$\Gamma_{C\rightarrow 2\pi} = \frac{m_C}{80}\frac{\chi_{C\pi\pi}{}^2}{4\pi}\left(\frac{m_C}{m_\pi}\right)^2\left(1-\frac{4m_\pi{}^2}{m_C{}^2}\right)^{5/2}. \quad \text{(A3)}$$

Similarly, we find for the decay $C \rightarrow K+\bar{K}$ the rate

$$\Gamma_{C\rightarrow K+\bar{K}} = \frac{m_C}{60}\frac{\chi_{CKK}{}^2}{4\pi}\left(\frac{m_C}{m_K}\right)^2\left(1-\frac{4m_K{}^2}{m_C{}^2}\right)^{5/2}. \quad \text{(A4)}$$

If universality is approximately valid, $\chi_{C\pi\pi}(m_C/m_\pi) \sim \chi_{CKK}(m_C/m_K) \sim \chi_{CG}$, and the decay rates $\Gamma_{C\rightarrow\pi^++\pi^-}$ and $\Gamma_{C\rightarrow K^++K^-}$ differ only by the phase space factor. Thus, for the ratio of the 2π and $2K$ rates, we find

$$\Gamma_{C\rightarrow\pi^++\pi^-}/\Gamma_{C\rightarrow K^++K^-} = 9.6 \quad \text{if } m_C = 1.25 \text{ GeV}$$
$$= 2.6\times10^3 \quad \text{if } m_C = 1.00 \text{ GeV}, \quad \text{(A5)}$$

independent of the coupling constant χ_{CG}.

We can now utilize the fact that the spin-2 resonance lies on a Regge trajectory to arrive at a crude estimate for its width. The contribution of the Pomeranchuk trajectory to the $T=0$ $\pi\pi$ scattering amplitude is[1,4,15]

$$T = \left(\frac{s}{s_0}\right)^{\alpha_P}\left[\frac{1+e^{-i\pi\alpha_P(t)}}{2\sin\pi\alpha_P(t)}\right]2s_0\eta_{P\pi\pi}{}^2\alpha_P(t)\,, \quad \text{(A6)}$$

where we have defined the coupling $\eta_{P\pi\pi}$ so that the ghost suppression factor $\alpha_P(t)$ is explicit in the amplitude. At t equal to the energy of the resonance, the Regge pole residue can be identified with the coupling constant of the unstable particle to two pions;

$$2\frac{\eta_{P\pi\pi}{}^2(m_C{}^2)}{\pi\epsilon_C}\frac{2m_\pi{}^2}{s_0} = \chi_{C\pi\pi}{}^2\,, \quad \text{(A7)}$$

where $\epsilon_C = d\,(\text{Re}\alpha_P)/dt$ at $t = m_C{}^2$.

From the optical theorem, the asymptotic total $\pi\pi$ cross section is equal to $\eta_{P\pi\pi}{}^2(0)$. It is thus apparent that to make an estimate of the decay rate, one must know the quantity $2[\eta_{P\pi\pi}{}^2(m_C{}^2)/\eta_{P\pi\pi}{}^2(0)](\epsilon_C s_0)^{-1}$, to which the decay rate is proportional. Our estimate is crude because we assume that the above factor is 2, which we believe not to be off by an order of magnitude. From the analysis of the differential cross section for elastic nucleon-nucleon scattering, we know that

$$\epsilon_P = d(\text{Re}\alpha_P(t))/dt|_{t=0} \approx 1 \text{ (GeV)}^{-2}\,,$$

and for the nucleon-nucleon channel we have found from our data analysis that $s_0 \approx 1$ GeV^2. It is our conjecture that the scaling factor s_0 is a property of the trajectory, rather than a different parameter for each reaction. The last consideration in making our assumption is the expectation, or hope, that the functions $\eta_{P\pi\pi}(t)$ and $d(\text{Re}\alpha_P(t))/dt$ do not change drastically between $t=0$ and $t=m_C{}^2$.

From the factorization theorem, and the asymptotic πN and NN total cross sections, it follows that $\sigma_{\pi\pi} = \eta_{P\pi\pi}{}^2(0) \approx 12$ mb.[20] Inserting this into Eq. (A7), we are led to

$$\chi_{C\pi\pi}{}^2/4\pi \approx m_\pi{}^2\sigma_{\pi\pi}/2\pi^2 = 0.03\,. \quad \text{(A8)}$$

This value gives the following decay rates:

$$\Gamma_{C\rightarrow 2\pi} = 32 \text{ MeV} \quad \text{for} \quad m_C = 1.0 \text{ GeV}$$
$$= 66 \text{ MeV} \quad \text{for} \quad m_C = 1.25 \text{ GeV}\,,$$
$$\Gamma_{C\rightarrow K\bar{K}} = 10 \text{ keV} \quad \text{for} \quad m_C = 1.0 \text{ GeV}$$
$$= 8.9 \text{ MeV} \quad \text{for} \quad m_C = 1.25 \text{ GeV}.$$

It is interesting to note that the universality concept formulated by Freund[34] is identical to ours, but that the consequences for the meson-meson, meson-baryon, and baryon-baryon total cross sections are different. In both papers, the coupling of the spin-2 particle to another object is proportional to the mass of the object. However, if the pole term has the Regge character, the coupling at $t=0$, which regulates the asymptotic cross section, is proportional to $s_0{}^{1/2}\chi_{CG}/m_C$. Freund essentially assumes that $s_0{}^{1/2}$ is reaction-dependent, and proportional to the mass of the particle to which the Regge pole is coupled, whereas we assume that the scaling factor s_0 is a constant for each trajectory.

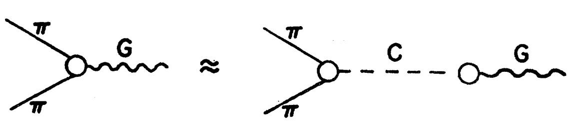

FIG. 3. Dominance of gravitational form factor by C meson.

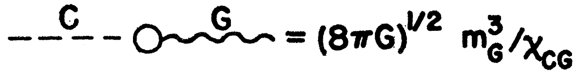

FIG. 4. Definition of graviton-C-meson coupling.